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General Certificate of Education January 2007 Advanced Level Examination

MATHEMATICS Unit Pure Core 4

MPC4

Thursday 25 January 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

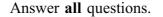
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.



1 A curve is defined by the parametric equations

$$x = 1 + 2t$$
, $y = 1 - 4t^2$

(a) (i) Find
$$\frac{dx}{dt}$$
 and $\frac{dy}{dt}$. (2 marks)

(ii) Hence find
$$\frac{dy}{dx}$$
 in terms of t. (2 marks)

(b) Find an equation of the normal to the curve at the point where t = 1. (4 marks)

- (c) Find a cartesian equation of the curve. (3 marks)
- 2 The polynomial f(x) is defined by $f(x) = 2x^3 7x^2 + 13$.
 - (a) Use the Remainder Theorem to find the remainder when f(x) is divided by (2x 3). (2 marks)
 - (b) The polynomial g(x) is defined by $g(x) = 2x^3 7x^2 + 13 + d$, where d is a constant. Given that (2x - 3) is a factor of g(x), show that d = -4. (2 marks)
 - (c) Express g(x) in the form $(2x-3)(x^2+ax+b)$. (2 marks)
- 3 (a) Express $\cos 2x$ in terms of $\sin x$. (1 mark)
 - (b) (i) Hence show that $3\sin x \cos 2x = 2\sin^2 x + 3\sin x 1$ for all values of x. (2 marks)
 - (ii) Solve the equation $3\sin x \cos 2x = 1$ for $0^\circ < x < 360^\circ$. (4 marks)
 - (c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. (2 marks)

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www.mymainscloud.com (i) Express $\frac{3x-5}{x-3}$ in the form $A + \frac{B}{x-3}$, where A and B are integers. (a) 4

(ii) Hence find
$$\int \frac{3x-5}{x-3} dx$$
. (2 marks)

(b) (i) Express
$$\frac{6x-5}{4x^2-25}$$
 in the form $\frac{P}{2x+5} + \frac{Q}{2x-5}$, where P and Q are integers.
(3 marks)

(ii) Hence find
$$\int \frac{6x-5}{4x^2-25} dx$$
. (3 marks)

5 (a) Find the binomial expansion of
$$(1+x)^{\frac{1}{3}}$$
 up to the term in x^2 . (2 marks)

(b) (i) Show that
$$(8+3x)^{\frac{1}{3}} \approx 2 + \frac{1}{4}x - \frac{1}{32}x^2$$
 for small values of x. (3 marks)

(ii) Hence show that
$$\sqrt[3]{9} \approx \frac{599}{288}$$
. (2 marks)

The points A, B and C have coordinates (3, -2, 4), (5, 4, 0) and (11, 6, -4) respectively. 6

(a) (i) Find the vector
$$B\dot{A}$$
. (2 marks)

- (ii) Show that the size of angle *ABC* is $\cos^{-1}\left(-\frac{5}{7}\right)$. (5 marks)
- The line *l* has equation $\mathbf{r} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$. (b)
 - Verify that C lies on l. (2 marks) (i)
 - Show that AB is parallel to l. (1 mark) (ii)
- The quadrilateral ABCD is a parallelogram. Find the coordinates of D. (3 marks) (c)

Turn over for the next question

7 (a) Use the identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$.

(b) Show that

$$2 - 2\tan x - \frac{2\tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of x, $\tan 2x \neq 0$.

- 8 (a) (i) Solve the differential equation $\frac{dy}{dt} = y \sin t$ to obtain y in terms of t. (4 marks)
 - (ii) Given that y = 50 when $t = \pi$, show that $y = 50e^{-(1 + \cos t)}$. (3 marks)
 - (b) A wave machine at a leisure pool produces waves. The height of the water, y cm, above a fixed point at time t seconds is given by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y\sin t$$

- (i) Given that this height is 50 cm after π seconds, find, to the nearest centimetre, the height of the water after 6 seconds. (2 marks)
- (ii) Find $\frac{d^2y}{dt^2}$ and hence verify that the water reaches a maximum height after π seconds. (4 marks)

END OF QUESTIONS

(4 marks)

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(2 marks)